Optimal Control of Variable Stiffness Actuator for a Catching Task

By

Ajinkya Arun Bhole

Department of Mechanical Engineering
Birla Institute of Technology and Science, Pilani, Rajasthan

A report submitted to Birla Institute of Technology and Science, Pilani, Rajasthan in accordance with the requirements of the degree of B.E. Hons in Mechanical Engineering.

May 2016

Certificate

It is certified that the work contained in this report, titled **"Optimal Control of Variable Stiffness Actuator for a Catching Task"** by **Ajinkya Bhole**, has been carried out under my supervision and is to my satisfaction.

Date　　　　　　　　　　　　　　　　　　　　Dr. Arun Kumar Jalan

Optimal Control of Variable Stiffness Actuator for a Catching Task

Abstract

This work is a study of variable stiffness actuators and their optimal control strategies. If we look at the physical reasons and control principles that govern movements of humans and animals, we can see that variable stiffness forms a cardinal part of their actuation system and the control strategies emerging from the first principles of optimality result in their tasks being carried out efficiently.

This study provides a review of why and where are variable stiffness actuation strategies necessary, their physical realization methods and finally their optimal control. A simulation of a ball catching task is studied using a 1 degree of freedom link integrated with a variable stiffness actuator. Various optimal control problems are studied, namely, Brachistochrone, Energy-maximization and Effort-reduction problem. This work also contains a control task taking care of a noisy sensor input for the velocity of the ball.

Acknowledgements

I would like to take this opportunity to express profound gratitude and deep regards to my advisor **Dr. Arun Kumar Jalan** for his exemplary guidance, monitoring and constant encouragement throughout the course of this work. The help and guidance given by him time to time shall carry me a long way in the journey of life in which I am about to embark. Working on this project was an extremely challenging and interesting adventure.

Contents

List of Figures

Chapter 1

Introduction

Figure 1.1: Tasks involving Variable Stiffness Actuation

If we consider our daily tasks like throwing, running or hitting (Figure 1.1), we can observe that variable stiffness actuation plays an important role in carrying out our tasks efficiently. Taking biomechanics as an inspiration, roboticists are trying to integrate the mechanical systems of the robots with Variable Stiffness Actuators.

As of late, robots are being produced that can cooperate with people and participate with them [13]. Such robots must meet altogether different prerequisites than robots that work in processing plant situations. Specifically, when robots work in a changing environment where people are likewise present (Figure 1.2), the most essential necessity for such robots is that they are sheltered, both towards human and robots. This, thus, requires these robots should be versatile, so they can securely collaborate with situations with shifting properties and attributes.

Initially, robots have most often been position-controlled. This is sufficient as long as the joint trajectories of the robot are known in advance, which is generally the case in production lines. However, there are many applications in which position control does not suffice. In particular, when a robot needs to physically interact with the environment. For example, consider a robot with a sanding tool that needs to smoothen a particular surface of an object. In this scenario, the surface to be

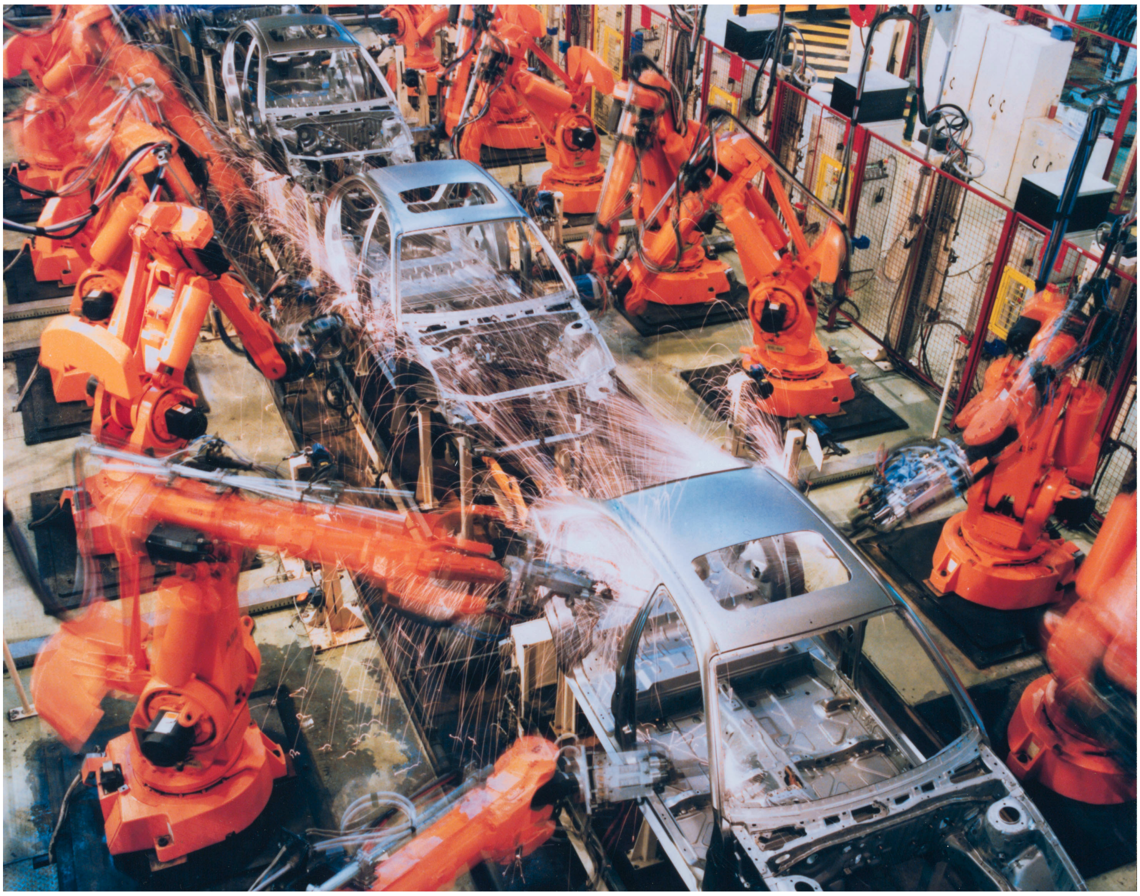

Figure 1.2: ABB-Spot Welding Robots. Robots are regularly utilized as a part of plants, where they perform repetitive assignments with high exactness at rapidity. However, their size, speed and their absence of natural mindfulness, makes such robots greatly perilous. *Image Source: abb.com*

.

sanded is inherently unknown. If a standard PID controller was applied, a static error due to the fact that the sanding tool cannot move through the surface to be sanded, might result in excessively high torques. This can be tackled through Force control strategy, where instead of defining the position trajectory for the tool, a desired interaction force is defined. Force control using Series elastic actuation [10] has shown many advantages over software emulated compliance. However, this impedance is constant, which implies that a robot equipped with series elastics actuators will need to be designed keeping in mind the properties of the environment it will be operating in. If the robot is to operate in a wide variety of environments with different properties, it might not be possible to find a fixed actuator impedance that is suitable for all scenarios. This is were Variable Stiffness Actuators offer a solution.

Chapter 2

Variable Stiffness Actuators

2.1 What are Variable Stiffness Actuators?

2.1.1 Stiff Actuators

The monotonous and high-exactness nature of industrial tasks has resulted in the design of highly specialized robots, with stiff actuation to achieve the required levels of precision. Stiff Actuation implies that the combination of the motor and motor control result in a joint that appears to be stiff, i.e. when disturbed, the joint motion deviates very little from the desired motion.

2.1.2 Series Elastic Actuators (SEA)

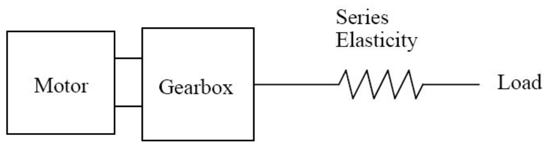

Figure 2.1: Series Elastic Actuator

These type of actuators (Figure 2.1) mainly allow to control the interaction forces between the robot and the environment, which is an essential ability for robots operating in an unknown environment [10].

2.1.3 Variable Stiffness Actuators (VSA)

The defining characteristic of variable stiffness actuators (Figure 2.2) is that this class of actuators is capable of varying the apparent output stiffness independently of the actuator output position. The variable stiffness characteristic allows greater versatility in tasks than SEAs.

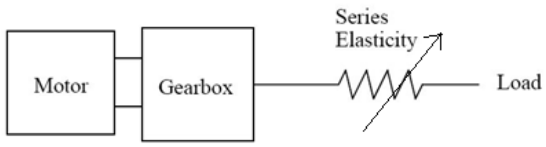

Figure 2.2: Variable Stiffness Actuator

2.2 Physical Realization of VSAs

Researchers have come up with ingenious designs for the physical realization of VSAs, many of which can be classified in the following categories based on their working principles. [5] provides a brief review of the state of the art Compliant Actuators.

2.2.1 Antagonistic Spring Setup (Biomorphic Design)

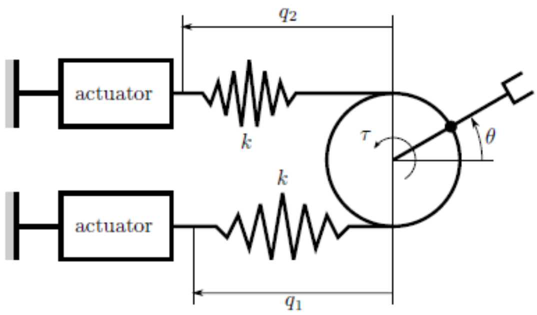

Figure 2.3: Antagonistic Setup

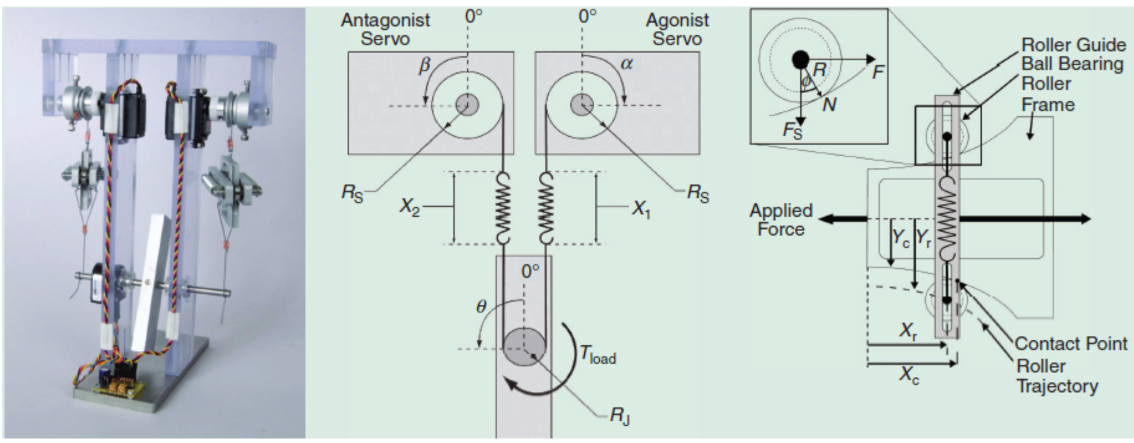

Figure 2.4: Antagonistic Variable Stiffness Actuation Setup [**bio**]

Main advantage of antagonistic setup is the intuitive design and the simple construction principle (Figure 2.3). The main drawbacks are that the range of output stiffness that can be achieved is proportional to the operating range of the internal degrees of freedom. Furthermore, if the stiffness is increased, work is converted in potential energy stored in the springs. This energy is essentially locked up, and cannot be used to do useful work at the output [11]. An example of antagonistically controlled VSA is shown in Figure 2.4.

2.2.2 Pretension Mechanisms

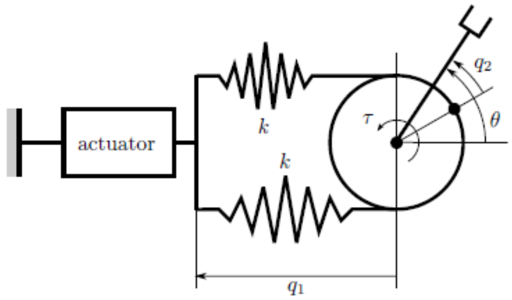

Figure 2.5: Pretension Mechanism Setup

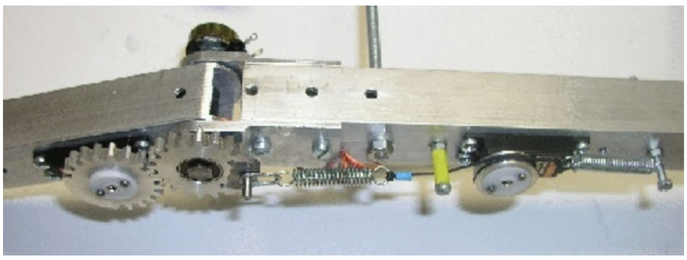

Figure 2.6: Pretension Mechanism Variable Stiffness Actuation Setup [**sethu**]

In the antagonistic spring setup, both internal degrees of freedom are used equivalently to vary the apparent output stiffness and to change the equilibrium position of the actuator. This means that, if for a particular set of design requirements it is needed to vary the stiffness over only a small range, while the equilibrium position needs to be varied over a large range, both internal degrees of freedom need to have a large range of motion to accommodate both requirements. This drawback can be overcome by decoupling stiffness control and equilibrium position control (Figure 2.5). In this design, specific requirements for the range of stiffness and the range of motion of the actuator can be met by using adequate motors for the actuation of the corresponding degrees of freedom. However, this design still suffers from locked up energy. An example of antagonistically controlled VSA is shown in Figure 2.6.

2.2.3 Variable Transmission Ratio

Instead of changing the apparent actuator output stiffness by modifying the state of the internal springs, as is done in the previous two designs, the apparent stiffness can also be changed by introducing a variable transmission ratio between the internal spring element and the actuator output (Figure 2.7), thus avoiding the potential lock-up of energy. This concept is essentially an evolution of the series elastics actuation principle. An example of variable transmission ratio controlled VSA is shown in Figure 2.8.

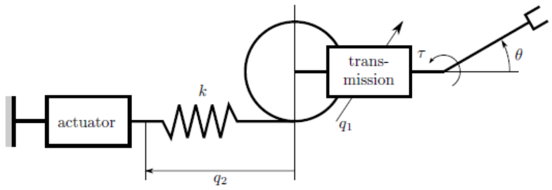

Figure 2.7: Variable Transmission Ratio Setup

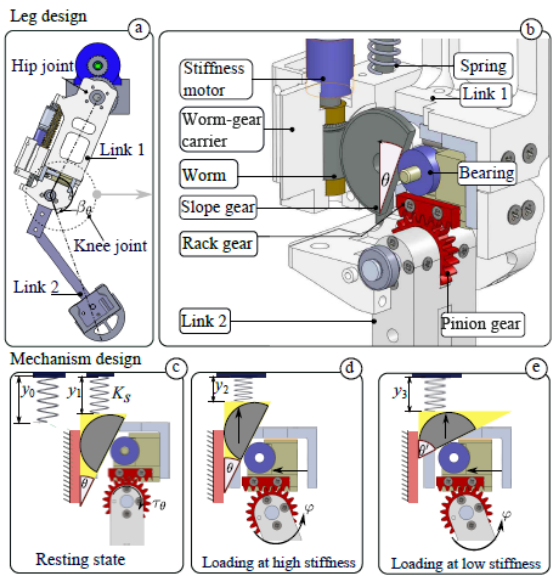

Figure 2.8: Variable Transmission Ratio Variable Stiffness Actuation Setup [**vtrr**]

There are many other design which do not fall under any of the above categories. Some of these include changing the active length of the spring [9] or controlling the apparent actuator stiffness by changing the configuration of a set of permanent magnets inside the actuator.

Chapter 3

Control Frameworks

Many researchers have come up with various control strategies for VSA. Some are as follows:

- **Feedback Linearisation**: A viable nonlinear control technique for solving trajectory tracking problems [3].
- **Passivity-based Control framework** :Control technique for torque, position, as well as impedance control of flexible joint robots [1].
- **Constraint-based Control**: Places constraints on the commands sent to the VSA to ensure that the equilibrium position and stiffness of the VSA is tracked to the desired values [7].
- **Optimal Control**: In case the desired trajectory of the equilibrium position and the stiffness values is not known, Optimal Control can come to the rescue. A performance index needs to be defined for the system which then takes care of the trajectory generation [4].

The next section discusses the optimal control problem for Variable Stiffness Actuators.

Chapter 4

Optimal Control

"Since the fabric of the universe is most perfect and the work of a most wise Creator, nothing at all takes place in the universe in which some rule of maximum or minimum does not appear..."

– Leonhard Euler

4.1 Problem Definition

Researchers have shown that VSAs can perform more economically in many tasks than SEAs and Stiff actuation. Some of these tasks form a part of safe physical Human Robot Interaction (pHRI), Energy transfer maximization, minimizing control efforts, etc. An optimal control problem involves a performance index which needs to be minimized/maximized over system and some desired constraints. In general any performance index can be stated by the equation below, which involves the terminal cost and the running cost.

$$
\begin{aligned}
\underset{u(t),t_f,p,x_0}{\text{minimize}} \quad & J(x(.), u(.), p, t_f) \\
\text{subject to} \quad & \dot{x} = f(x(t), u(t), p, t), x(t_0) = x_0 \\
& g_L \leq g(x(t), u(t), p, t) \leq g_U \\
& \phi_L \leq \phi(x_0, x_f, u_0, u_f, p, t_f) \leq \phi_U \\
& x_L \leq x(t) \leq x_U \\
& u_L \leq u(t) \leq u_U \\
& p_L \leq p \leq p_U
\end{aligned}
$$

where, $u_0 \triangleq u(t_0)$, $x_f \triangleq x(t_f)$ and $u_f \triangleq u(t_f)$. Here the cost function is defined as

$$J(x(.), u(.), p, t_f) \triangleq \int_{t_0}^{t_f} L(x(t), u(t), p, t)dt + E(x_0, x_f, u_0, u_f, p, t_f)$$

Some of the well known optimal control problems are as follows:

4.2 Safe Brachistochrone Problem

The safe brachistochrone control problem can commonly be seen in safe-Physical Human Robot Interaction situations, where constraint is put over robot velocities. Following is the problem statement:

PROBLEM STATEMENT

$$\begin{cases} \min_T \int_0^T 1 \, dt & \text{COST INDEX} \\ M_{\text{rot}} \ddot{x}_{\text{rot}} + u_K (x_{\text{rot}} - x_{\text{link}}) = u_{\text{act}} & \text{ROTOR DYNAMICS} \\ M_{\text{link}} \ddot{x}_{\text{link}} + u_K (x_{\text{link}} - x_{\text{rot}}) = 0 & \text{LINK DYNAMICS} \\ |\dot{x}_{\text{link}}| \leq v_{\text{safe}} & \text{SAFETY CONSTRAINT} \\ |u_{\text{act}}| \leq U_{\max} & \text{ACTUATOR TORQUE LIMIT} \\ u_{K,\min} \leq u_K \leq u_{K,\max} & \text{STIFFNESS RANGE} \end{cases}$$

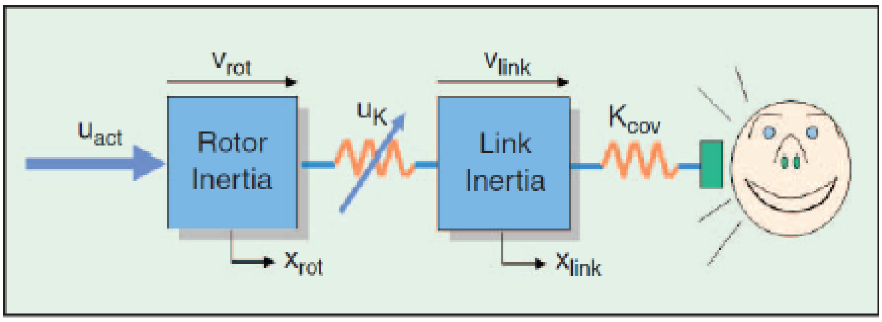

Figure 4.1: The Brachistochrone Problem statement

4.3 Energy Transfer Maximization Problem

The Energy Transfer Maximization Problem can commonly be seen in tasks where energy of the system at the terminal time needs to be maximized. Following is the problem statement:

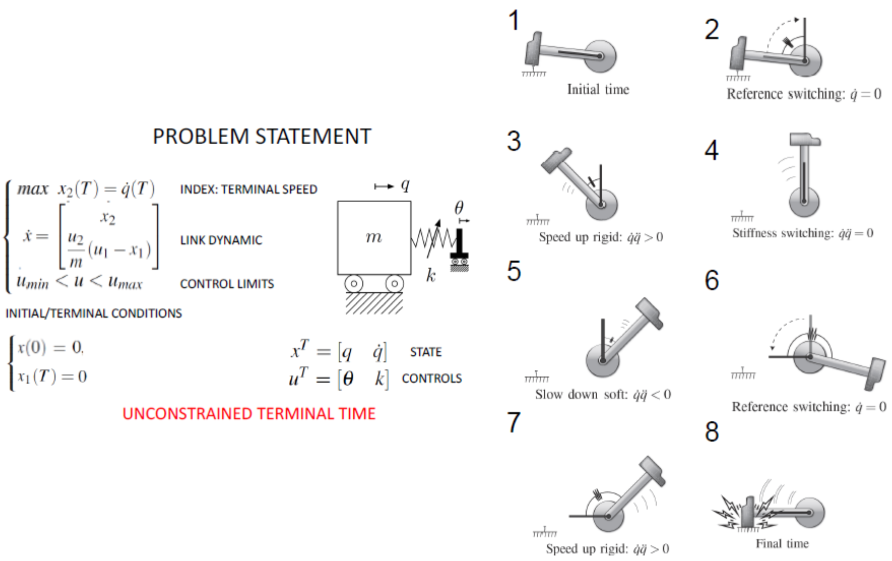

Figure 4.2: The Energy Transfer Maximization problem statement

4.4 Effort Minimization Problem

The Effort Minimization Problem can commonly be seen in tasks where the running effort cost needs to be minimized. Following is the problem statement:

$$J_w = -d(\mathbf{q}(T), \dot{\mathbf{q}}(T)) + \frac{w}{2} \int_0^T \|\mathbf{F}(\mathbf{q}, \mathbf{u})\|^2 \, dt$$

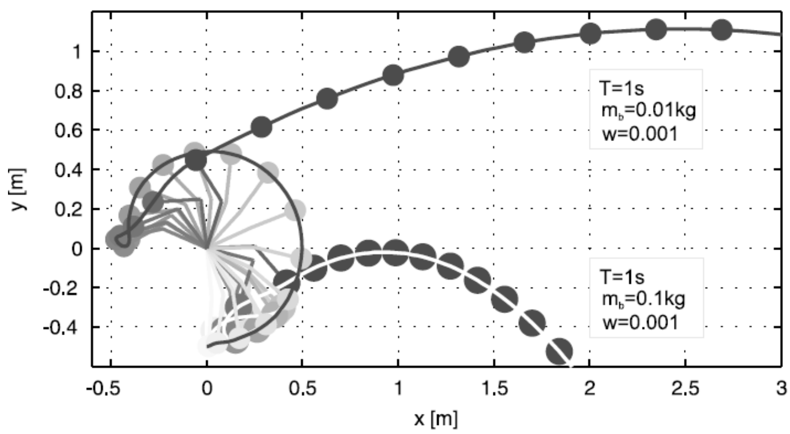

Figure 4.3: The Effort Minimization Problem statement

(The images in this section had the following Image Source: http://www.centropiaggio.unipi.it/)

5

Optimal Control of a Ball Catching Task

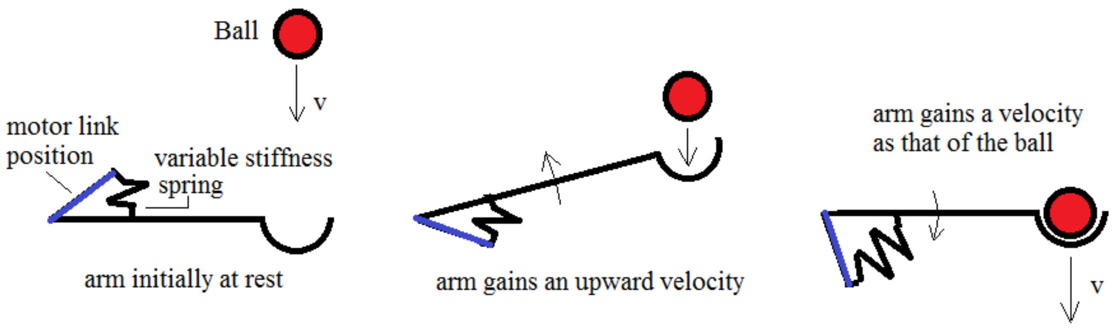

Figure 5.1: A ball catching problem using compliant actuation

This chapter contains the formulation of the model for a ball catching task and its possible optimal control problems. As shown in Figure 5.1, a simple 1 DOF arm with compliant actuation is considered. The following sections consider various control problems and finally compares them. All the optimal control problems were solved using TOMLAB's `propt` solver [6].

5.1 Brachistochrone Problem Using Series Elastic Actuation

In this task, a compliant element with fixed stiffness is added to the actuator. Following are the state space equations of the model:

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \qquad u = \theta$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{I} & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$i.e. \quad \dot{x} = Ax + Bu$$

where, x is the state of the system, q is the arm position, $\dot{(q)}$ is the arm velocity, u is the control input, θ is the motor position, k is the stiffness value of the compliant element and I is the link's rotational inertial.

5.1.1 Problem Definition

The objective is to attain a specific velocity in the minimum possible time, which can be stated as follows:

$$\begin{aligned} \underset{u(t),x_0}{\text{minimize}} \quad & t_f \\ \text{subject to} \quad & \dot{x} = Ax + Bu \\ & x_{10} = 0 \quad x_{20} = 0 \quad x_{1tf} = 0 \quad x_{2tf} = -1.5 \\ & -\frac{2\pi}{3} \leq x_1 \leq \frac{2\pi}{3} \\ & -2 \leq x_2 \leq 2 \\ & -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \\ & -5 \leq \dot{\theta} \leq 5 \end{aligned}$$

5.1.2 MATLAB CODE and Results

```
%% The Brachistochrone Problem

%% Problem setup
toms t
toms t_f
p = tomPhase('p', t, 0, t_f, 60);
setPhase(p);

tomStates x y theta
tomControls v
```

```

% Initial guess
% Note: The guess for t_f must appear in the list before
    expression involving t.
x0 = {t_f == 3
      icollocate({
      x == 0
      y == 0
      theta == 0
      })
      collocate(v==0.01)};

% Box constraints
cbox = {0.1 <= t_f <= 100
      -2*pi/3 <= icollocate(x) <= 2*pi/3
      -2 <= icollocate(y) <= 2
      -pi/2 <= icollocate(theta) <= pi/2
      -5 <= collocate(v) <= 5};

% Boundary constraints
cbnd = {initial({x == 0; y == 0; theta == 0})
      final({x == 0; y == -1.5})};

% ODEs and path constraints
ceq = collocate({
      dot(x) == y
      dot(y) == -0.5*x + theta
      dot(theta) == v});

% Objective
objective = t_f;

%% Solve the problem
options = struct;
options.name = 'Brachistochrone_SEA';
solution = ezsolve(objective, {cbox, cbnd, ceq}, x0, options
    );
```

```
%% Plot the result
% To obtain the brachistochrone curve, we plot y versus x.
subplot(4,1,1)
ezplot(t, x);
xlabel('Time (sec)') % x-axis label
ylabel('Link Pos(rad)') % y-axis label

subplot(4,1,2)
ezplot(t, y);
xlabel('Time (sec)') % x-axis label
ylabel('Link V(rad/sec)') % y-axis label

subplot(4,1,3)
ezplot(t, theta);
xlabel('Time (sec)') % x-axis label
ylabel('Mot-Pos(rad)') % y-axis label

subplot(4,1,4)
ezplot(t, v)
xlabel('Time (sec)') % x-axis label
ylabel('Mot-Vel(rad/sec)') % y-axis label
```

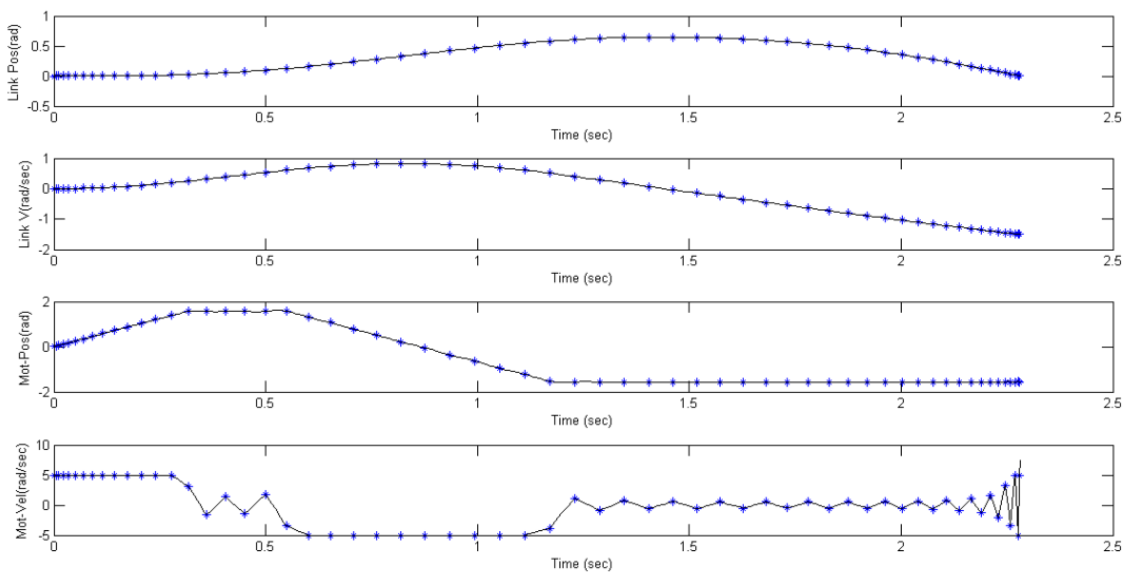

Figure 5.2: SEA Brachistochrone Problem Results

5.2 Brachistochrone Problem Using Variable Stiffness Actuation

In this task, a compliant element with variable stiffness is added to the actuator. Following are the state space equations of the model:

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \qquad u = \begin{bmatrix} \theta \\ k\dot{q} \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} x_2 \\ -\frac{u_2}{I}(u_1 - x_1) \end{bmatrix}$$

where, x is the state of the system, q is the arm position, $\dot{(q)}$ is the arm velocity, u is the control input, θ is the motor position, k is the stiffness value of the compliant element and I is the link's rotational inertial.

5.2.1 Problem Definition

The objective is to attain a specific velocity in the minimum possible time, which can be stated as follows:

$$\begin{aligned}
\underset{u(t),x_0}{\text{minimize}} \quad & t_f \\
\text{subject to} \quad & \dot{x_1} = x_2 \quad \dot{x_2} = -\frac{u_2}{I}(u_1 - x_1) \\
& x_{10} = 0 \quad x_{20} = 0 \quad x_{1tf} = 0 \quad x_{2tf} = -1.5 \\
& -\frac{2\pi}{3} \leq x_1 \leq \frac{2\pi}{3} \\
& -2 \leq x_2 \leq 2 \\
& -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \\
& -5 \leq \dot{\theta} \leq 5 \\
& 0 \leq k \leq 0.5
\end{aligned}$$

5.2.2 MATLAB CODE and Results

```
%% The Brachistochrone Problem —— VSA

%% Problem setup
toms t
```

```
toms t_f
p = tomPhase('p', t, 0, t_f, 50);
setPhase(p);

tomStates x y
tomControls theta k

% Initial guess
% Note: The guess for t_f must appear in the list before
    expression involving t.
x0 = {t_f == 3
     icollocate({
     x == 0
     y == 0
     })
     collocate({theta==0
     k==0
     })};

% Box constraints
cbox = {0.1 <= t_f <= 100
     -2*pi/3 <= icollocate(x) <= 2*pi/3
     -2 <= icollocate(y) <= 2
     -pi/2 <= collocate(theta) <= pi/2
     0 <= collocate(k) <= 0.5};

% Boundary constraints
cbnd = {initial({x == 0; y == 0; theta == 0})
     final({x == 0; y == -1.5})};

% ODEs and path constraints
ceq = collocate({
     dot(x) == y
     dot(y) == -k*x + theta*k
     });

% Objective
```

```
objective = t_f; %integrate(5*u.^2)

%% Solve the problem
options = struct;
options.name = 'SEA';
solution = ezsolve(objective, {cbox, cbnd, ceq}, x0, options
    );

%% Plot the result
% To obtain the brachistochrone curve, we plot y versus x.
subplot(4,1,1)
ezplot(t, x);
xlabel('Time (sec)') % x-axis label
ylabel('Link Pos(rad)') % y-axis label

subplot(4,1,2)
ezplot(t, y);
xlabel('Time (sec)') % x-axis label
ylabel('Link V(rad/sec)') % y-axis label

subplot(4,1,3)
ezplot(t, theta);
xlabel('Time (sec)') % x-axis label
ylabel('Mot-Pos(rad)') % y-axis label

% We can also plot theta vs. t.
subplot(4,1,4)
ezplot(t, k)
xlabel('Time (sec)') % x-axis label
ylabel('Stiff(N-m/rad)') % y-axis label
```

From Figure 5.2 and Figure 5.3 we can see how varying the stiffness helped in performing the task at a reduced cost i.e. reduced time.

5.3 Effort Minimization Problem using VSA

The system model for this problem remains the same as that for the Brachistochrone problem using VSA.

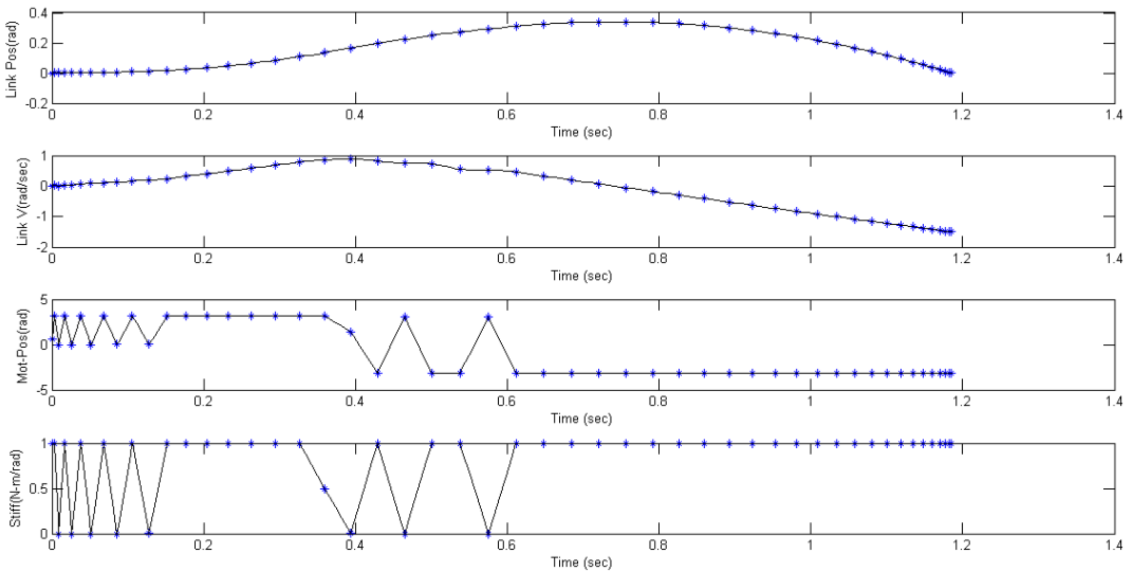

Figure 5.3: VSA Brachistochrone Problem Results

5.3.1 Problem Definition

We have a sensor providing the position and velocity of the ball. The objective is to shoulder the arm with a velocity equal to that of the ball at the moment the ball touches the arm. The sensor reading can be noisy. Therefore, the control system implemented should adapt to this noisy sensor, update its optimal control law and finally achieve the terminal desired speed. Figure 5.4 shows the implemented control system:

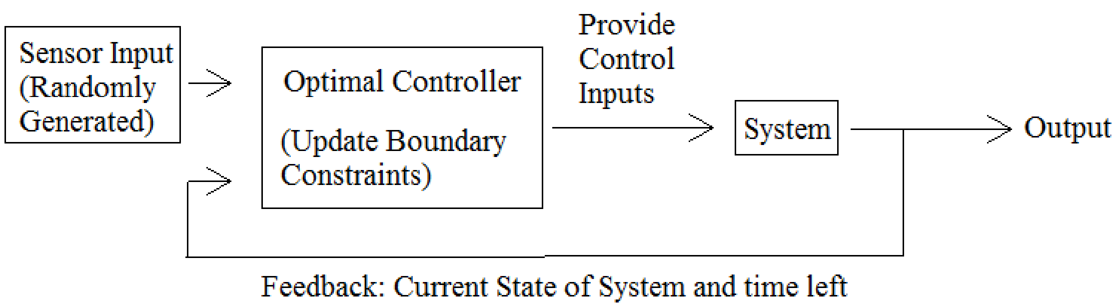

Figure 5.4: Control System to tackle noisy sensor readings

The problem can be stated as follows:

$$
\begin{aligned}
\underset{u(t),x_0}{\text{minimize}} \quad & \int_{t_0}^{t_f} u^2 + k^2 \\
\text{subject to} \quad & \dot{x_1} = x_2 \quad \dot{x_2} = -\frac{u_2}{I}(u_1 - x_1) \\
& x_{10} = 0 \quad x_{20} = 0 \quad x_{1tf} = 0 \quad x_{2tf} = -v_{sensor} \\
& -\frac{2\pi}{3} \le x_1 \le \frac{2\pi}{3} \\
& -2 \le x_2 \le 2 \\
& -\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \\
& -5 \le \dot{\theta} \le 5 \\
& 0 \le k \le 5
\end{aligned}
$$

5.3.2 MATLAB CODE and Results

```
% Generate Random Sensor Readings

up=1.7;low=1.3;fspeed=1.5;lastmany=2;
 tit=60;gap=1;tim=2;

 sense=(up-low).*rand(tit,1)+low;
 sense(tit-lastmany:tit,1)=fspeed;
answ=zeros(tit+1,4);
 time=zeros(tit+1,1);

% Time array
 for i = 1:tit
      time(i+1,1)=time(i,1)+(tim/tit);
end

% Optimal Controller
 for i = 1:tit-gap
      answ(i+1,:)=Opt(tim-(i-1)*(tim/tit),tit-(i-1),sense(i,1)
          ,answ(i,:));
end

a=[0;answ(1:tit+1-gap,1)];b=[0;answ(1:tit+1-gap,2)];c=[0;
    answ(1:tit+1-gap,3)];d=[0;answ(1:tit+1-gap,4)];
```

```
ttime=time(1:tit+1,1);
tatime=time(2:tit+1,1);
sense(tit-lastmany:tit,1)=fspeed;
sens=sense(1:tit+1-gap,1);
% Plot Results

subplot(5,1,1)
plot(ttime, a);
xlabel('Time (sec)') % x-axis label
ylabel('Link Pos(rad)') % y-axis label

subplot(5,1,2)
plot(ttime, b);
xlabel('Time (sec)') % x-axis label
ylabel('Link V(rad/sec)') % y-axis label

subplot(5,1,3)
plot(ttime, c);
xlabel('Time (sec)') % x-axis label
ylabel('Mot-Pos(rad)') % y-axis label

% We can also plot theta vs. t.
subplot(5,1,4)
plot(ttime, d)
xlabel('Time (sec)') % x-axis label
ylabel('Stiff(N-m/rad)') % y-axis label

subplot(5,1,5)
plot(tatime, sense)
xlabel('Time (sec)') % x-axis label
ylabel('SensorO/P(m/s)') % y-axis label
```

```
function soln=Opt(time,iter,speed,answ)

%% Problem setup
toms t
%toms t_f
p = tomPhase('p', t, 0, time, iter);
```

```
setPhase(p);

tomStates x y
tomControls theta k

% Initial guess
% Note: The guess for t_f must appear in the list before
    expression involving t.
x0 = {
      icollocate({
      x == answ(1,1)
      y == answ(1,2)
      })
      collocate({theta==answ(1,3)
      k==answ(1,4)
      })};

% Box constraints
cbox = {
      -2*pi/3 <= icollocate(x) <= 2*pi/3
      -2 <= icollocate(y) <= 2
      -pi <= collocate(theta) <= pi
      0 <= collocate(k) <= 5};

% Boundary constraints
cbnd = {initial({x == answ(1,1); y == answ(1,2); theta ==
    answ(1,3); k == answ(1,4)})
      final({x == 0; y == -speed})};

% ODEs and path constraints
ceq = collocate({
      dot(x) == y
      dot(y) == -k*x + theta*k
      });

% Objective
objective = integrate(0.5*theta.^2+0.5*k.^2);
```

```
%% Solve the problem
options = struct;
options.name = 'SEA';
solution = ezsolve(objective, {cbox, cbnd, ceq}, x0, options);

%% Provide solution

soln=[solution.x_p(3,1) solution.y_p(3,1) solution.theta_p(2,1) solution.k_p(2,1)];
```

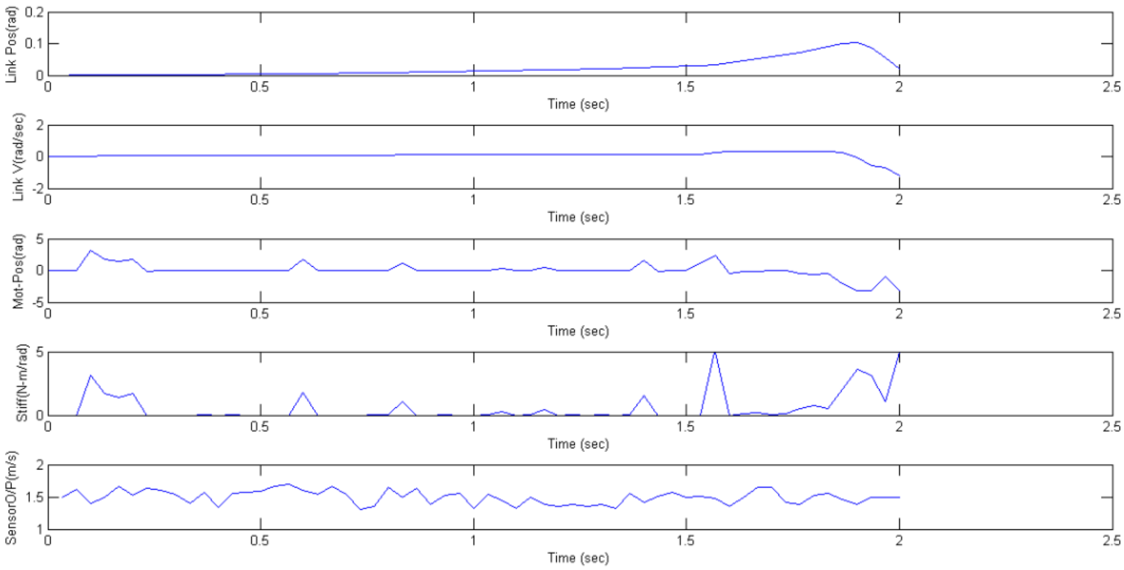

Figure 5.5: Control Effort Minimization Problem Results

6

Discussion

This work studied Variable Stiffness Actuators, their physical realization methodologies and optimal control problems.

Variable Stiffness actuation strategy has shown tremendous advantage over previous stiff actuation and series elastic actuation. Variable stiffness can be realized in mechanical systems using methods like antagonistic actuation, pretension mechanisms, variable transmission ratio mechanisms and many more.

One of the most emerging topics in the field of Variable Stiffness actuation is their control using the first principles of optimality. This work studied commonly known optimal control problems like safe brachistochrone, energy maximization and control effort reduction. Optimal control can prove advantageous in case the desired trajectory of the equilibrium position and stiffness is not known.

This work considered a ball catching task with a 1 DOF link with compliant actuation. The link was optimally controlled using a SEA and VSA. As expected, link with VSA could achieve the task much more efficiently. Lastly, an effort minimization task was considered. This task involved adapting to a noisy sensor reading. The implemented controller was able to provide the link with quite an accurate desired velocity at the terminal time.

Bibliography

[1] Alin Albu-Schäffer, Christian Ott, and Gerd Hirzinger. "A unified passivity-based control framework for position, torque and impedance control of flexible joint robots". In: *The International Journal of Robotics Research* 26.1 (2007), pp. 23–39.

[2] David Braun, Matthew Howard, and Sethu Vijayakumar. "Optimal variable stiffness control: formulation and application to explosive movement tasks". In: *Autonomous Robots* 33.3 (2012), pp. 237–253.

[3] Alessandro De Luca, Riccardo Farina, and Pasquale Lucibello. "On the control of robots with visco-elastic joints". In: *Robotics and Automation, 2005. ICRA 2005. Proceedings of the 2005 IEEE International Conference on*. IEEE. 2005, pp. 4297–4302.

[4] Manolo Garabini et al. "Optimality principles in variable stiffness control: The vsa hammer". In: *Intelligent Robots and Systems (IROS), 2011 IEEE/RSJ International Conference on*. IEEE. 2011, pp. 3770–3775.

[5] R v Ham et al. "Compliant actuator designs". In: *Robotics & Automation Magazine, IEEE* 16.3 (2009), pp. 81–94.

[6] Kenneth Holmström. "The TOMLAB optimization environment in Matlab". In: (1999).

[7] Matthew Howard, David J Braun, and Sethu Vijayakumar. "Constraint-based equilibrium and stiffness control of variable stiffness actuators". In: *Robotics and Automation (ICRA), 2011 IEEE International Conference on*. IEEE. 2011, pp. 5554–5560.

[8] Shane A Migliore, Edgar A Brown, and Stephen P DeWeerth. "Biologically inspired joint stiffness control". In: *Robotics and Automation, 2005. ICRA 2005. Proceedings of the 2005 IEEE International Conference on*. IEEE. 2005, pp. 4508–4513.

[9] Toshio Morita and Shigeki Sugano. "Design and development of a new robot joint using a mechanical impedance adjuster". In: *Robotics and Automation, 1995. Proceedings., 1995 IEEE International Conference on*. Vol. 3. IEEE. 1995, pp. 2469–2475.

[10] Gill A Pratt and Matthew M Williamson. "Series elastic actuators". In: *Intelligent Robots and Systems 95.'Human Robot Interaction and Cooperative Robots', Proceedings. 1995 IEEE/RSJ International Conference on*. Vol. 1. IEEE. 1995, pp. 399–406.

[11] Ludo Christian Visser. *Variable stiffness actuators: modeling, control, and application to compliant bipedal walking*. University of Twente, 2013.

[12] Hung Vu et al. "Improving energy efficiency of hopping locomotion by using a variable stiffness actuator". In: ().

[13] Martin Wassink and Stefano Stramigioli. "Towards a novel safety norm for domestic robotics". In: *Intelligent Robots and Systems, 2007. IROS 2007. IEEE/RSJ International Conference on*. IEEE. 2007, pp. 3354–3359.